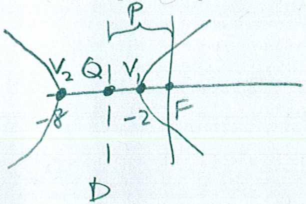
A hyperbola has a focus at the pole, and vertices with polar co-ordinates $(2, \pi)$ and $(8, \pi)$. **SCORE: _____ / 15 PTS**

Find a polar equation of the hyperbola.



$$\boxed{r = \frac{ep}{1 - e\cos\theta}} \quad 3$$

$$e = \frac{V_1F}{V_1Q} = \frac{V_2F}{V_2Q}$$

$$1\tfrac{1}{2} \quad = \boxed{\frac{2}{p-2}} = \boxed{\frac{8}{8-p}} \quad 1\tfrac{1}{2}$$

$$2(8-p) = 8(p-2)$$

$$16 - 2p = 8p - 16$$

$$32 = 10p$$

$$\boxed{p = \frac{16}{5}} \quad 1\tfrac{1}{2}$$

$$e = \frac{2}{\frac{16}{5} - 2} \cdot \frac{5}{5} = \frac{10}{16-10} = \boxed{\frac{5}{3}} \quad 1\tfrac{1}{2}$$

$$r = \boxed{\frac{\frac{\cancel{5}}{3} \cdot \frac{16}{\cancel{5}}}{1 - \frac{5}{3}\cos\theta}} \cdot \frac{3}{3} = \boxed{\frac{16}{3 - 5\cos\theta}}$$

$1\tfrac{1}{2}$ $\quad$ $1\tfrac{1}{2}$

Find the logarithmic formula for $\sinh^{-1} x$ by solving $x = \sinh y$ for y using the exponential definition and an algebraic substitution $z = e^y$.

SCORE: _____ / 20 PTS

$x = \sinh y$

$x = \boxed{\dfrac{e^y - e^{-y}}{2}} = \boxed{\dfrac{z - \frac{1}{z}}{2}}$ 3 3

$2x = z - \frac{1}{z}$

$2xz = z^2 - 1$

$\boxed{0 = z^2 - 2xz - 1}$ 4

$\boxed{z = \dfrac{2x \pm \sqrt{4x^2 + 4}}{2}}$ 4

$z = x \pm \sqrt{x^2 + 1}$ 3

$\boxed{e^y = x + \sqrt{x^2 + 1}}$ (SINCE $e^y > 0$) $\rightarrow$ $y = \underline{\ln(x + \sqrt{x^2+1})}$ 3

$= \sinh^{-1} x$

Convert the rectangular equation $x^2 + 6y - 9 = 0$ to polar. **SCORE: _____ / 15 PTS**

$$\boxed{r^2\cos^2\theta + 6r\sin\theta - 9 = 0} \quad 4\tfrac{1}{2}$$

$$\boxed{r = \frac{-6\sin\theta \pm \sqrt{36\sin^2\theta + 36\cos^2\theta}}{2\cos^2\theta}} \quad 4\tfrac{1}{2}$$

$$r = \frac{-6\sin\theta \pm \sqrt{36}}{2\cos^2\theta}$$

$$r = \frac{-6\sin\theta \pm \boxed{6}}{2\cos^2\theta} \; 1\tfrac{1}{2} = \frac{6(-\sin\theta \pm 1)}{\boxed{2(1-\sin^2\theta)}} \; 1\tfrac{1}{2} = \frac{3(-\sin\theta \pm 1)}{(1-\sin\theta)(1+\sin\theta)}$$

$$\boxed{r = \frac{3}{1+\sin\theta}} \quad 1\tfrac{1}{2}$$

$$\boxed{r = \frac{-3}{1-\sin\theta}} \quad 1\tfrac{1}{2}$$

Eliminate the parameter to find a rectangular equation corresponding to the parametric equations

SCORE: _____ / 15 PTS

$$x = \frac{1+t}{1-t}, \qquad y = \frac{t+2}{t-2}$$

Write your final answer in the form y as a simplified function in terms of x.

$x(1-t) = 1+t$

$\boxed{x - xt = 1+t}$

$x - 1 = t + xt$

$\boxed{x-1 = t(1+x)}$

$\boxed{t = \frac{x-1}{1+x}}$

$\boxed{y = \frac{\frac{x-1}{1+x} + 2}{\frac{x-1}{1+x} - 2}} \cdot \frac{1+x}{1+x}$

$\boxed{y = \frac{x-1+2+2x}{x-1-2-2x}}$

$\boxed{y = \frac{3x+1}{-x-3}} = -\frac{3x+1}{x+3}$

$2\frac{1}{2}$ EACH

AK and BK were working on their polar graphing partner quiz. **SCORE: _____ / 35 PTS**

On the question about the polar equation $r = 6 + 4\sqrt{3}\sin 3\theta$, they determined correctly that the symmetry tests $(-r, \pi - \theta)$, $(-r, \theta)$, $(-r, -\theta)$ and $(r, \pi + \theta)$ do **NOT** indicate that the graph is symmetric.

AXIS POLE $\theta = \frac{\pi}{2}$ POLE

[a] **Using their results, along with the tests and shortcuts shown in lecture**, test if the graph is symmetric over the pole, the polar axis and/or $\theta = \frac{\pi}{2}$. State your conclusions in the table. **NOTE: Run as FEW tests as needed to prove your answers are correct.**

AXIS: $r = 6 + 4\sqrt{3}\sin 3(-\theta)$ 3

$r = 6 - 4\sqrt{3}\sin 3\theta$ 2 CAN'T TELL

$\theta = \frac{\pi}{2}$: $r = 6 + 4\sqrt{3}\sin 3(\pi - \theta)$ 3

$r = 6 + 4\sqrt{3}\sin(3\pi - 3\theta)$

$r = 6 + 4\sqrt{3}(\sin 3\pi \cos 3\theta - \cos 3\pi \sin 3\theta)$ (0, −1)

$r = 6 + 4\sqrt{3}\sin 3\theta$ SYM 3

Type of symmetry	Conclusion
Over the pole	CAN'T TELL
Over the polar axis	CAN'T TELL
Over $\theta = \frac{\pi}{2}$	SYM

2 IF 2 CORRECT
4 IF ALL 3 CORRECT

[b] Based on the results of part [a], what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph) ?

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ 5

[c] Find all angles **algebraically** in the minimum interval in part [b] at which the graph goes through the pole.

$0 = 6 + 4\sqrt{3}\sin 3\theta$ 2

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\sin 3\theta = -\frac{6}{4\sqrt{3}} = -\frac{6\sqrt{3}}{12} = -\frac{\sqrt{3}}{2}$ 5

$-\frac{3\pi}{2} \le 3\theta \le \frac{3\pi}{2}$

$3\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{4\pi}{3}$ 5

$\theta = -\frac{2\pi}{9}, -\frac{\pi}{9}, \frac{4\pi}{9}$ 3

Name the shape of the graphs of the following polar equations.
If the graph is a rose curve, state the number of petals.

SCORE: _____ / 10 PTS

[a] $r = 3\cos 4\theta$

2 2

ROSE CURVE, 8 PETALS

[b] $r = \dfrac{1}{4 - 3\cos\theta} = \dfrac{\frac{1}{4}}{1 - \frac{3}{4}\cos\theta}$

ELLIPSE 3

[c] $r = 3 + 4\sin\theta$ $\left|\frac{3}{4}\right| < 1$

LIMACON WITH LOOP 3

Find parametric equations for the line through the points $(-15, 5)$ and $(-2, -9)$. **SCORE: _____ / 10 PTS**

Your answer should not use either $x = t$ nor $y = t$.

$x = -15 + (-2 - {}^-15)t \longrightarrow x = -15 + 13t$ 5

$y = 5 + (-9 - 5)t \longrightarrow y = 5 - 14t$ 5

OR

$x = -2 + ({}^-15 - {}^-2)t \longrightarrow x = -2 - 13t$

$y = -9 + (5 - {}^-9)t \longrightarrow y = -9 + 14t$

Let $\sinh x = -7$. **SCORE: _____ / 20 PTS**

[a] Find the value of $\coth x$ using identities. (You do NOT need to prove the identities you use.)
NOTE: Your solution must NOT use inverse hyperbolic functions nor their logarithmic formulae.

4 $\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x - 49 = 1$

3 $\cosh^2 x = 50$

2 $\cosh x = 5\sqrt{2}$ (SINCE $\cosh x > 0$ FOR ALL $x \in \mathbb{R}$)

$\coth x = \dfrac{\cosh x}{\sinh x} = -\dfrac{5\sqrt{2}}{7}$ 4

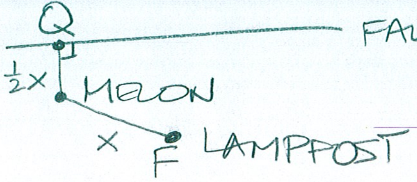
[b] Find the value of $\sinh 2x$ using identities. (You do NOT need to prove the identities you use.)
NOTE: Your solution must NOT use inverse hyperbolic functions nor their logarithmic formulae.

$\sinh 2x = 2 \sinh x \cosh x = 2(-7)(5\sqrt{2}) = -70\sqrt{2}$

4 3

As a little boy, Melon Husk lived on Falcon Ave (a straight road). He would ride his Stella tricycle on a path so that his distance from Falcon Ave was always half his distance from a certain lamppost on the sidewalk. What was the shape of the path on which Melon rode his tricycle ? **Draw a diagram and write algebraic equations involving distances to justify your answer.**

SCORE: _____ / 10 PTS



4

$$e = \frac{MF}{MQ} = \frac{x}{\frac{1}{2}x} = 2 > 1$$

4

HYPERBOLA

2